Reality isn’t what it seems

Bell’s Theorem for Everyone

Vikram Vyas

Outline

- Einstein: Reality and its relationship with physical theories
- Quantum description of an electrons spin
- Quantum description of two electrons: Entanglement
- EPR “Paradox”: Einstein - Bohr Debate
- Bell’s Contribution: Turning a philosophical debate into an experiment question
- A more powerful and easier to understand version of Bell’s theorem by: Greenberger, Horne, Shimony and Zeilinger
- Conclusion?

Where it all began

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Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

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In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

1.

ANY serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.

In attempting to judge the success of a physical theory, we may ask ourselves two questions: (1) "Is the theory correct?" and (2) "Is the description given by the theory complete?" It is only in the case in which positive answers may be given to both of these questions, that the concepts of the theory may be said to be satisfactory. The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of experiment and measurement. It is the second question that we wish to consider here, as applied to quantum mechanics.

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory.* We shall call this the condition of completeness. The second question is thus easily answered, as soon as we are able to decide what are the elements of the physical reality.

The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It seems to us that this criterion, while far from exhausting all possible ways of recognizing a physical reality, at least provides us with one

EPR: The Philosophical Point of View

Relationship between the physical world and its description by our theories

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EPR: Criterion of a successful theory

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EPR: What is a Complete Theory

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EPR: Physical Reality

The Two main points: David Bohm’s analysis (from his book QM)

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- If, without in any way disturbing the system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to this physical quantity.

    A. The world can correctly be analyzed in terms of distinct and separately existing "elements of reality.”

    B. Every one of these elements must be a counterpart of a precisely defined mathematical quantity appearing in a complete theory.

Quantum Mechanics - 1

Bare minimum needed to understand EPR

- A state of the system is described by a **vector** in an **abstract** space

$$|\psi >$$

- Since a state is a vector therefore if $|\phi >$ is another possible state of the system then

$$c_1|\psi > +c_2|\phi >$$

  is also a possible state of the system.

- The states encode the observable properties of the system in their behavior under linear maps.

- For every observable property **S** of the system there is a map $\hat{S}$ and special states with the property

$$\hat{S}|S >= s|S >$$

  where s is the value of the property **S** as measured in an experiment.

Quantum Mechanics - 2

Spin half particles

- To describe the spin along the z axis for a particle like an electron, we have a map $\hat{S}_z$ and **two** special states

$$\hat{S}_z|\uparrow>=\frac{1}{2}\hbar|\uparrow>$$
$$\hat{S}_z|\downarrow>=-\frac{1}{2}\hbar|\downarrow>$$

- Any other state can be written as

$$|\psi>=c_1|\uparrow>+c_2|\downarrow>$$

- If we measure the spin along z axis then there is $|c_1|^2$ probability that it will be $\frac{1}{2}\hbar$ and $|c_2|^2$ probability that it will be $-\frac{1}{2}\hbar$.

Quantum Mechanics - 3

Entangled states

State vector of a "molecule" with zero spin that is made of two spin-half "atoms" **A** and **B** :

$$|E>= \frac{1}{\sqrt{2}} (| \uparrow>_A | \downarrow>_B - | \downarrow>_A | \uparrow>_B)$$

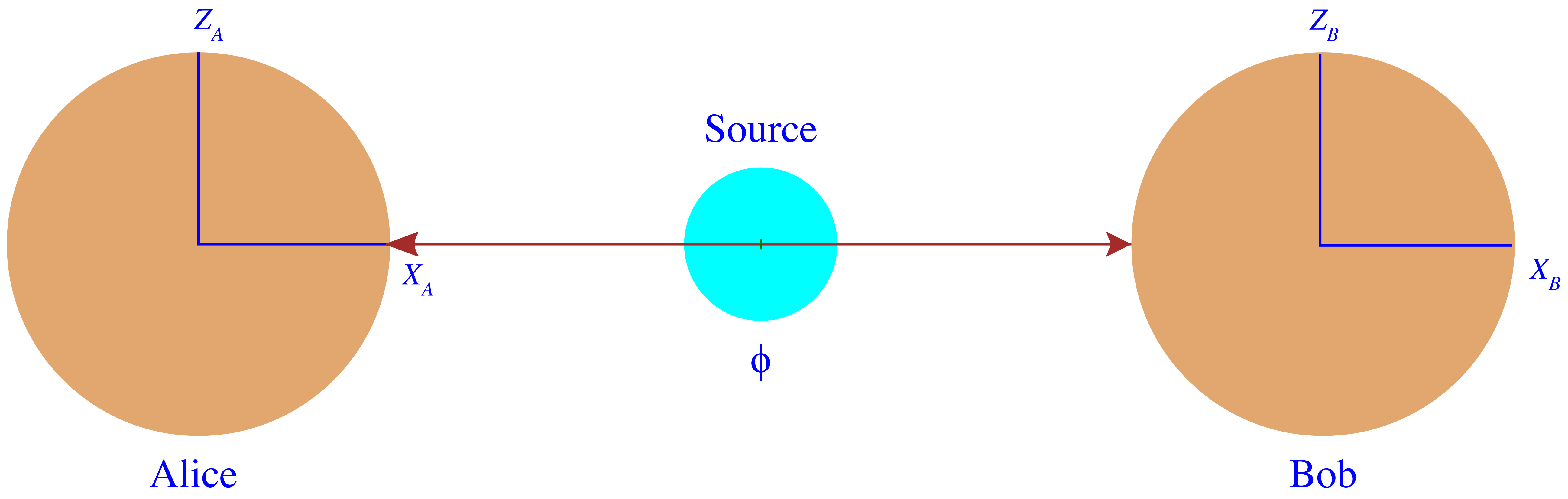
- A general state vector of the moleculue is a linear combination of pair of vectors

$$\{| \uparrow>_A | \uparrow>_B, | \uparrow>_A | \downarrow>_B, | \downarrow>_A | \uparrow>_B, | \downarrow>_A | \downarrow>_B\}$$

- The molecule in state $|E>$ has a well defined value of spin along z direction, which is zero but the atom **A** and **B** do **not** have a definite value of spin along z direction.

EPR Experiment

Bohm's Version

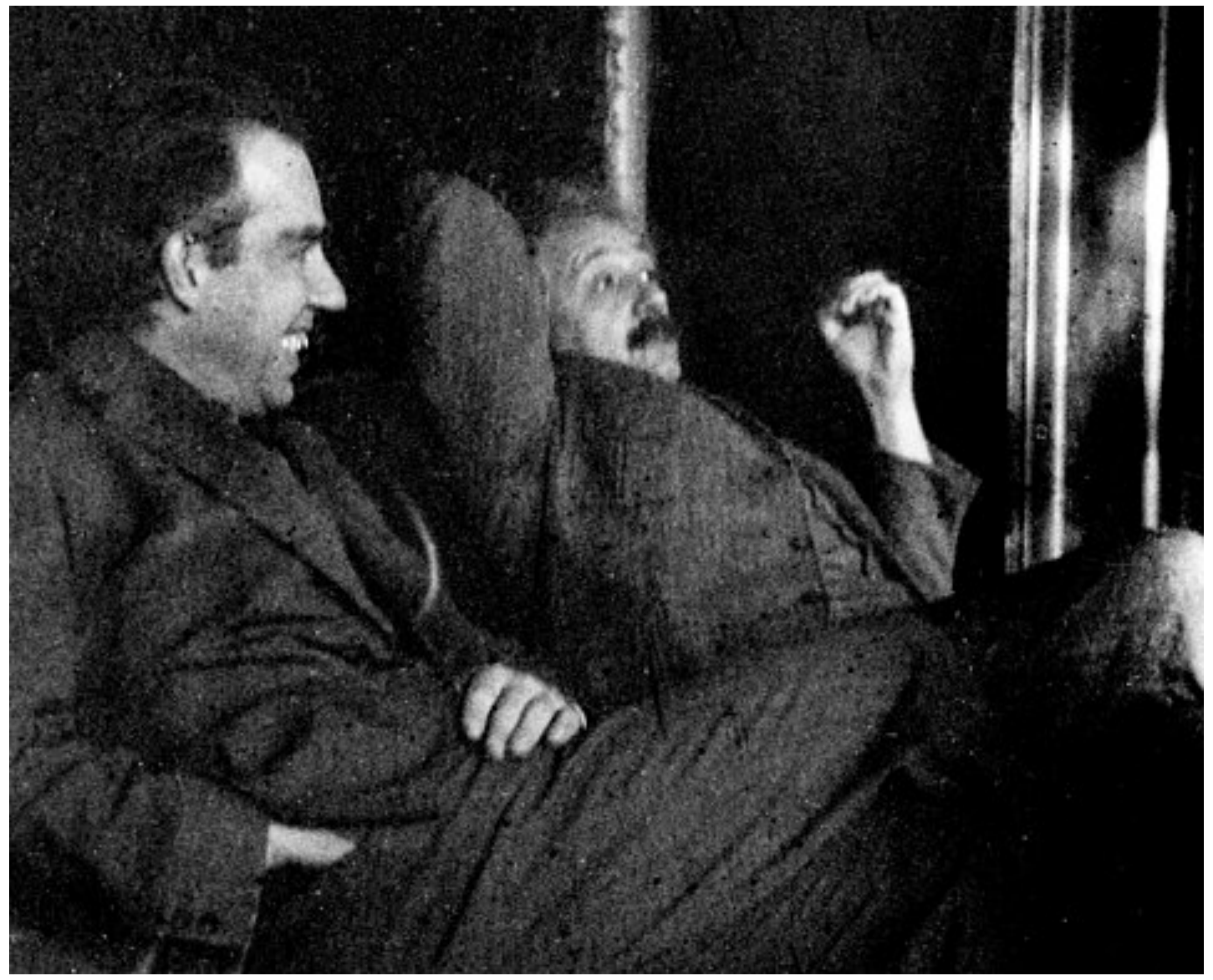


$$|E> = \frac{1}{\sqrt{2}}(|\uparrow>_A|\downarrow>_B - |\downarrow>_A|\uparrow>_B)$$

$$|E> = \frac{1}{\sqrt{2}}(|+\hat{x}>_A|-\hat{x}>_B - |-\hat{x}>_A|+\hat{x}>)$$

- If you measure particle A and find it spinning "down," particle B will **definitely** be spinning "up"
- If you measure particle A and find it spinning "up," particle B will **definitely** be spinning "down"
- If you measure particle A and find it spinning "right," particle B will definitely be spinning "left"
- If you measure particle A and find it spinning "left," particle B will definitely be spinning "right"

Einstein Bohr Debate - 1



$$|E> = \frac{1}{\sqrt{2}}(|\uparrow>_A|\downarrow>_B - |\downarrow>_A|\uparrow>_B)$$
$$|E> = \frac{1}{\sqrt{2}}(|+\hat{x}>_A|-\hat{x}>_B - |-\hat{x}>_A|+\hat{x}>)$$

If one measures the spin of particle **A**, far from the decay point, and finds spin up, say, then one knows with certainty that particle **B**, which is far away, has spin down.

Einstein

Since **A** has in no way disturbed particle **B**, then this feature, spin down, must be an element of physical reality. Therefore having spin down is a property of the particle **B** itself, and cannot have been produced by any measurement we made on particle **A** . Therefore **B** must have come away from the point of interaction, the decay point, with spin down.

Similarly if we measure the spin along x axis them **B** must also have come with the spin along the plus or minus x axis. Therefore spin along x axis too is an element of reality, and **B** must have come with a definite value of spin along x axis.

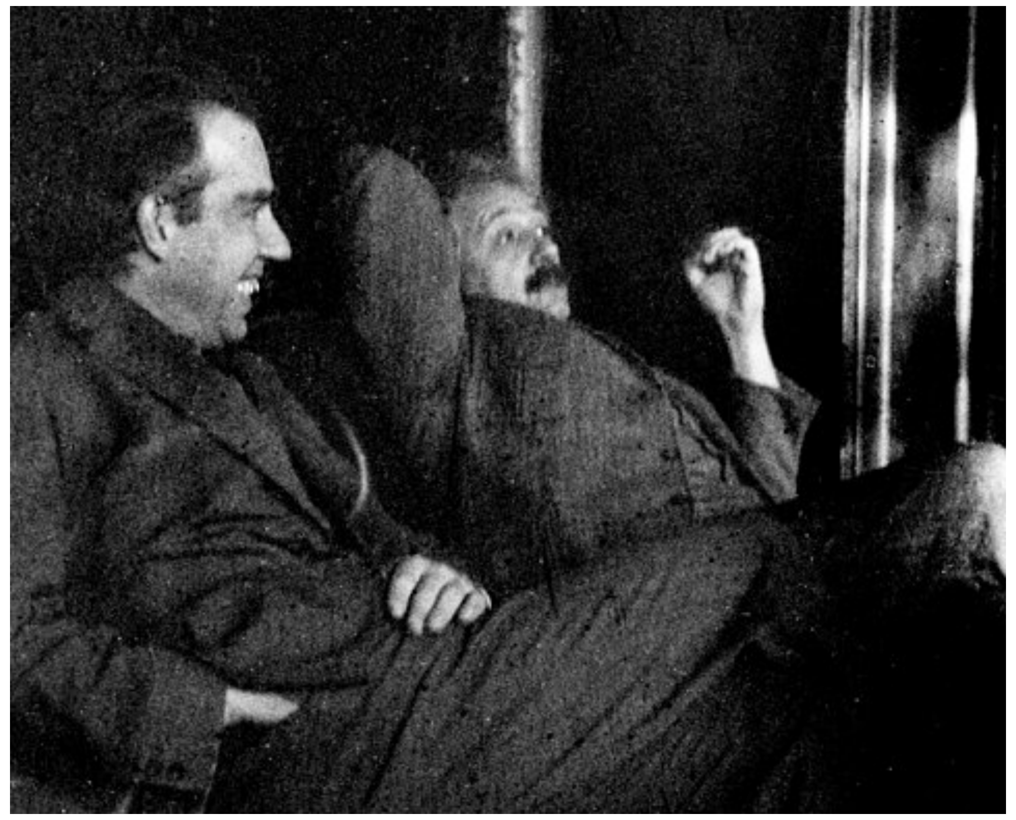
But quantum mechanical description has **no** state in which particle has a definite value of spin along z axis and spin along x axis. The quantum mechanical description is **incomplete**

Bohr

The spin of particle **B** is *indeterminate* until the spin of particle **A** is measured, as until then it was in a superposition of states up and down, and one could in principle have interference between the possibilities.

After the measurement the superimposed state collapsed into the state with with **A** up and **B** down.

Einstein Bohr Debate - 2



Einstein to Bohr

"Spooky action at distance"

But still there has to be a causal mechanism ....

Bohr to Einstein

No, its true that the superposition extends indefinitely but measurements by **B** will only reveal a statistical distribution, (in this case 50% up 50% down), but this statistical distribution is **independent of what A does**, so it cannot be used to send any signal. Its only when **A** and **B** meet that they will find there is a correlation in their measurements.

Causality is "unspeakable " in quantum mechanics

Just a Philosophical Debate?

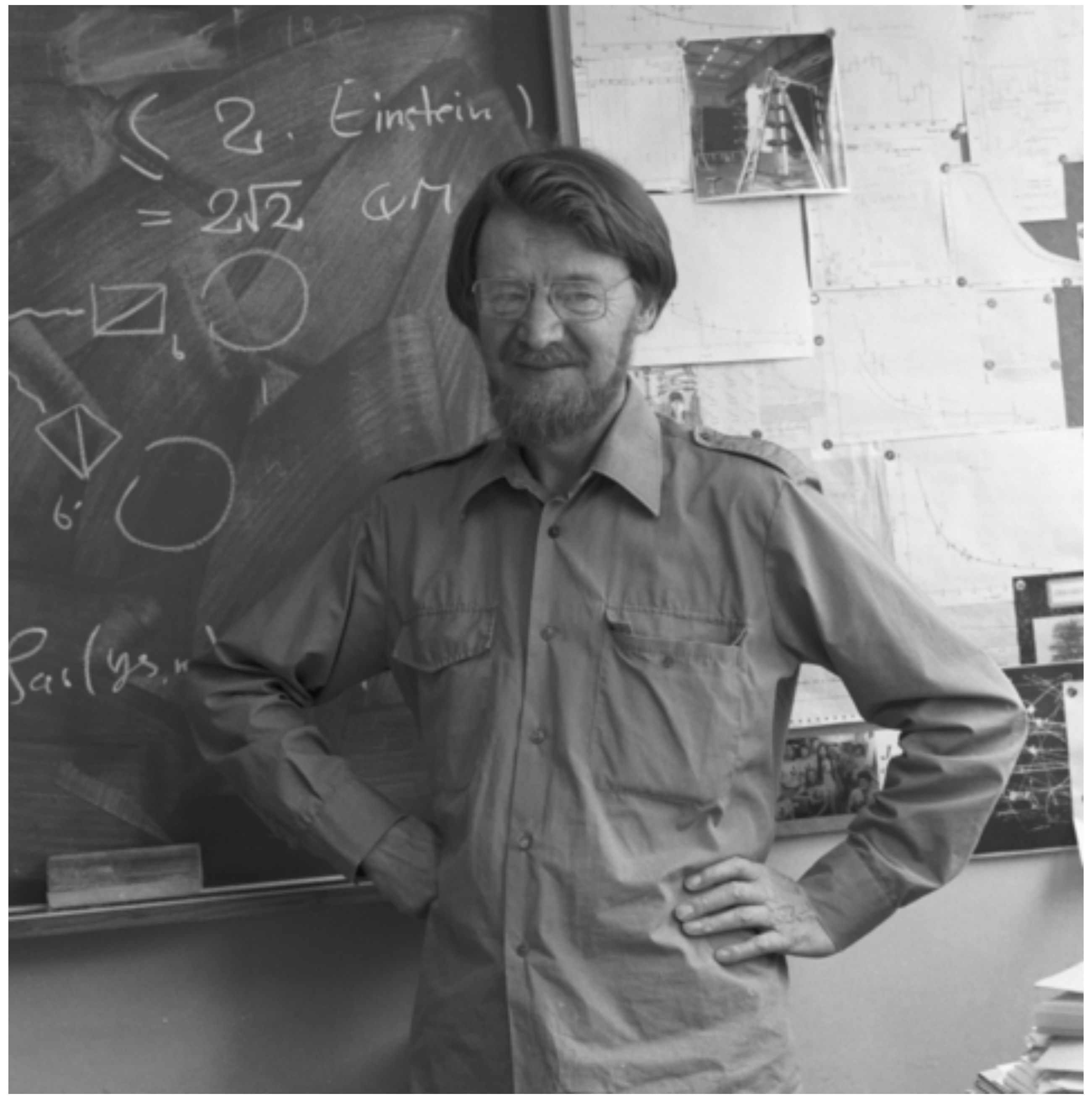
The difference between the two points of view had no experimental consequences.

Bell's theorem changed this ...

Readings

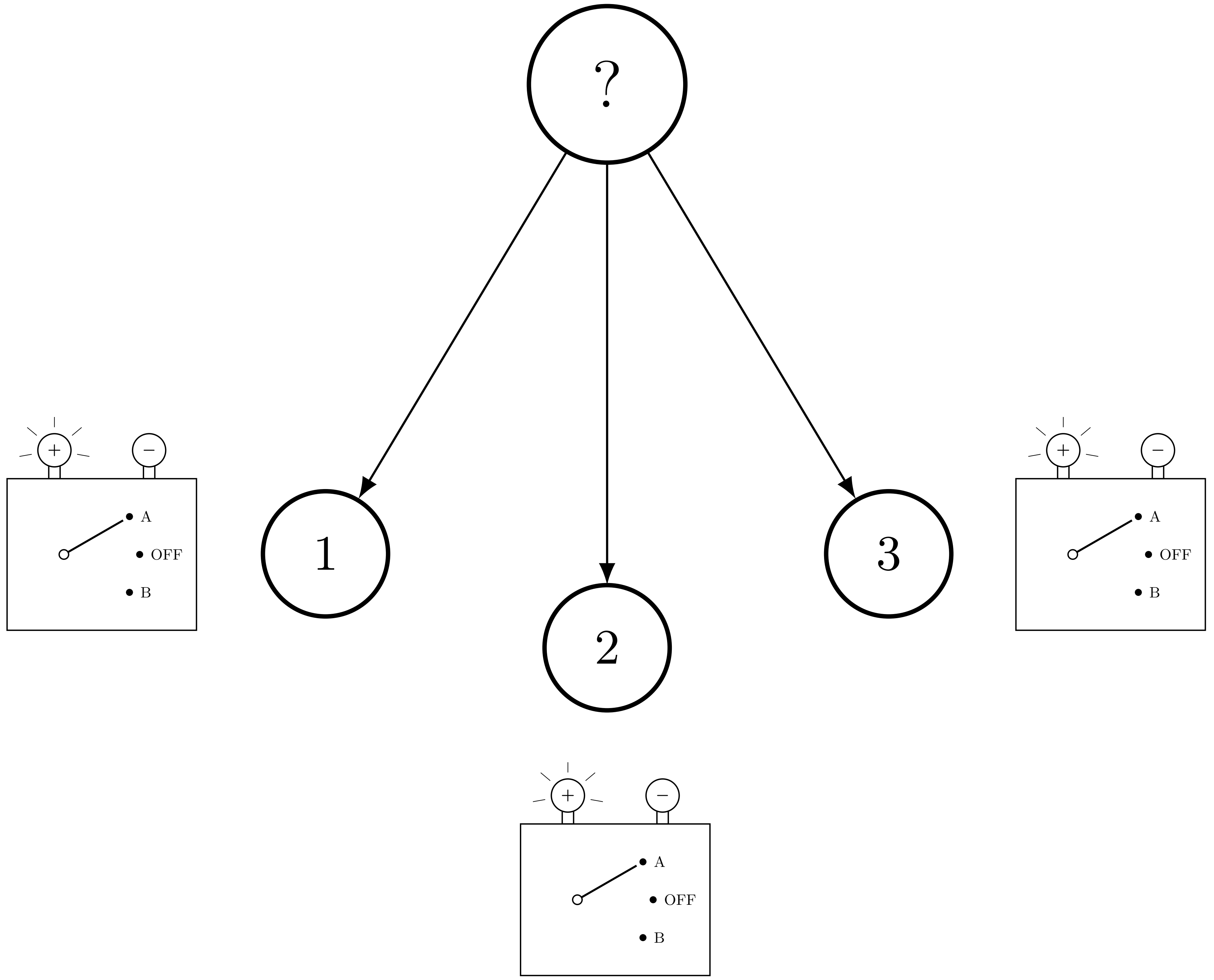
- **John S. Bell**, *Speakable and Unspeakable in Quantum Mechanics: Collected Papers on Quantum Philosophy.* Cambridge University Press, 1987 (2nd ed. 2004).
  - Speakable and Unspeakable in Quantum Mechanics
  - Bertlmann's Socks and the Nature of Reality

  These two essays for laypeople are strongly recommended. Bell was deeply influenced by Einstein's view of reality and wanted to confirm that the view was correct, but Bell's own work proved that the universe as we know it, does not follow Einstein's idea of reality. His essays are poignant and lyrical and are great examples of how science is practiced by some of the most original thinkers of our time.



Going Beyond Bell's Theorem

A more powerful and easier to understand version : GHZ and GHSZ
Greenberger, Horne, Zeilinger, and Greenberger, Horne, Shimony, Zeilinger
Coleman's version



Result of the Experiment

The result of the experiments done millions of time revel that whenever one A and two B's are measured the result is always:

$$A_1 B_2 B_3 = +1$$
$$B_1 A_2 B_3 = +1$$
$$B_1 B_2 A_3 = +1$$

Realist Analysis

The experimental results allows us to **predict** the value of property A and B of any of the particles without disturbing it. These properties are the **elements of reality**.

According to the realistic interpretation the experiments are reveling the **preexisting** properties of the quantities A and B **carried** by the **particles**. Therefore

$$(A_1 B_2 B_3)(B_1 A_2 B_3)(B_1 B_2 A_3) = +1$$
$$A_1 B_2 B_2 B_3 B_3 A_2 B_1 B_1 A_3 = +1$$
$$A_1 (B_2)^2 (B_3)^2 A_2 (B_1)^2 A_3 = +1$$

Using $(B_1)^2 = (B_2)^2 = (B_3)^2 = 1$ realists make a prediction:

$$\boxed{A_1 A_2 A_3 = +1}$$

A mathematical interlude

Some properties of the spin map $\hat{S}_i = \frac{1}{2}\hbar\hat{\sigma}_i$

- $$(\hat{\sigma}_x)|\uparrow>=|\downarrow>$$
- $$(\hat{\sigma}_x)|\downarrow>=|\uparrow>$$
- $$(\hat{\sigma}_y)|\uparrow>=i|\downarrow>$$
- $$(\hat{\sigma}_y)|\downarrow>=-i|\uparrow>$$

The Experiment: QM Analysis

$$|\psi>= \frac{1}{\sqrt{2}}(|\uparrow>_1|\uparrow>_2|\uparrow>_3 - |\downarrow>_1|\downarrow>_2|\downarrow>)$$

Three particles are in a superposition where either **all three spin up** or **all three spin down** - with opposite signs (phases) for the two possibilities. Measuring any one particle instantly determines the other two.

$A_1 \longleftrightarrow (\hat{\sigma_x})_1$ map related to the observed property of spin along x axis of particle 1

$B_1 \longleftrightarrow (\hat{\sigma_y})_1$ map related to the observed property of spin along y axis of particle 1

similarly for particle 2 and particle 3.

One can show that

$\hat{A}_1\hat{B}_2\hat{B}_3|\psi>= +1|\psi> \Rightarrow$ $|\psi>$ is a state in which $A_1B_2B_3$ has definite value $+1$

$\hat{B}_1\hat{A}_2\hat{B}_3|\psi>= +1|\psi> \Rightarrow$ $|\psi>$ is a state in which $B_1A_2B_3$ has definite value $+1$

$\hat{B}_1\hat{B}_2\hat{A}_3|\psi>= +1|\psi> \Rightarrow$ $|\psi>$ is a state in which $B_1B_2A_3$ has definite value $+1$

and

$$\hat{A}_1\hat{A}_2\hat{A}_3|\psi>= \underbrace{-1}_{\text{measured value}} |\psi>$$

therefore QM predicts

$$A_1A_2A_3 = -1$$

The Results of the Experiment

$$A_1 A_2 A_3 = -1$$

Farewell elements of reality!

Conclusion?

Its Quantum all the way?

Wittgenstein standing on a street corner lost in thought, and said: "What's bothering you, Ludwig?" Wittgenstein says: "I was just wondering why people said it was natural to believe the sun went around the earth rather than the other way around". The friend says: "Well, that's because it looks like the Sun goes around the earth". Wittgenstein thinks for a moment and says: "Tell me: What would it have looked like if it had looked like it was the other way around?"

"Still, the philosophy of Zurek's approach seems right to me … why not patiently and carefully work through what standard quantum mechanics can say about how information regarding a quantum object gets out into the observable world? Here the quantum pioneers left a lot of work unfinished in the revolution they started a century ago, prematurely foreclosing the issue (usually by insisting on the Copenhagen interpretation or just accepting it without question). Now we can at least hope to complete that task."

Philip Ball Quanta Article